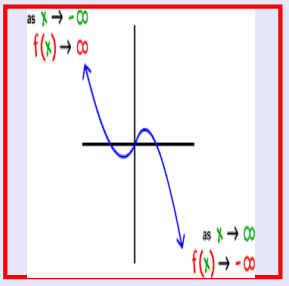
Math 245

Spring 2022

Lecture 23



Given $z = -4 + 2i$ $\quad z = a + bi \rightarrow |z| = \sqrt{a^2 + b^2}$

1) Real part -4 $\quad$ 2) Imaginary Part 2

3) $|z| = \sqrt{(-4)^2 + 2^2} = \sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4}\sqrt{5} = \boxed{2\sqrt{5}}$

$\bar{z}$ → Conjugate of z

$z = a + bi \rightarrow \bar{z} = a - bi$

4) $\bar{z} = \boxed{-4 - 2i}$

$z = 4 - 3i$

1) Re. Part = $\boxed{4}$

2) Im. Part = $\boxed{-3}$

3) $|z| = \sqrt{4^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = \boxed{5}$

4) $\bar{z} = \overline{4-3i} = \boxed{4+3i}$

5) $z \cdot \bar{z} = (4-3i)(4+3i) = 16 + 12i - 12i - 9i^2$

$= 16 - 9(-1)$

$= 16 + 9 = \boxed{25}$

Simplify

Recall:
$i^2 = -1$
$(-1)^{\text{even}} = 1$
$(-1)^{\text{odd}} = -1$

1) $i^{200} = [i^2]^{100}$

$= (-1)^{100} = \boxed{1}$

2) $i^{350} = (i^2)^{175}$

$= (-1)^{175} = \boxed{-1}$

3) $i^{175} = i^{174} \cdot i = [i^2]^{87} \cdot i = (-1)^{87} \cdot i = (-1)i = \boxed{-i}$

Given $z = 3 - 5i$, $w = -2 + 3i$

Find

1) $z + w$

$= 3 - 5i + (-2 + 3i)$

$= 3 - 5i - 2 + 3i = \boxed{1 - 2i}$

2) $z - w = 3 - 5i - (-2 + 3i)$

$= 3 - 5i + 2 - 3i$

$= \boxed{5 - 8i}$

3) $z \cdot w = (3 - 5i)(-2 + 3i)$

$= -6 + 9i + 10i - 15i^2$

$= -6 + 19i - 15(-1) = -6 + 19i + 15$

$= \boxed{9 + 19i}$

$\boxed{a + bi}$

How to divide by a Complex Number:

$$\frac{w}{z} = \frac{w \cdot \bar{z}}{z \cdot \bar{z}} = \boxed{a + bi}$$

$$\frac{2i}{1 - 3i} = \frac{2i(1 + 3i)}{(1 - 3i)(1 + 3i)} = \frac{2i + 6i^2}{1 + 3i - 3i - 9i^2}$$

$$= \frac{2i + 6(-1)}{1 - 9(-1)} = \frac{-6 + 2i}{1 + 9}$$

$$= \frac{-6 + 2i}{10} = \frac{-6}{10} + \frac{2}{10}i$$

$$= \boxed{-\frac{3}{5} + \frac{1}{5}i}$$

Re. Part $= -\frac{3}{5}$

Im. Part $\frac{1}{5}$

Divide $\dfrac{1-2i}{3+4i}$

$$\frac{1-2i}{3+4i} = \frac{(1-2i)(3-4i)}{(3+4i)(3-4i)} = \frac{3-4i-6i+8i^2}{9-12i+12i-16i^2}$$

$$= \frac{3-10i+8(-1)}{9-16(-1)}$$

$$= \frac{3-10i-8}{9+16} = \frac{-5-10i}{25}$$

$$= \frac{-5}{25} - \frac{10}{25}i$$

$$= \boxed{\frac{-1}{5} - \frac{2}{5}i}$$

Re. $\frac{-1}{5}$ Im. $\frac{-2}{5}$

Given $z = -8 - 6i$

1) $\bar{z} = \overline{-8-6i} = \boxed{-8+6i}$

2) $|z| = \sqrt{(-8)^2+(-6)^2}$

$= \sqrt{64+36}$

$= \sqrt{100} = \boxed{10}$

3) $\dfrac{1}{z} = \dfrac{1(-8+6i)}{(-8-6i)(-8+6i)}$

$$= \frac{-8+6i}{64-48i+48i-36i^2} = \frac{-8+6i}{64-36(-1)} = \frac{-8+6i}{64+36}$$

$$= \frac{-8+6i}{100} = \frac{-8}{100} + \frac{6}{100}i = \boxed{-.08+.06i}$$

$$= \boxed{\frac{-2}{25} + \frac{3}{50}i}$$

$w = 2 - 3i$ $\quad z = -2 + 4i$

Find

1) $z^2 = (-2+4i)^2$

$= (-2+4i)(-2+4i)$

$= 4 - 8i - 8i + 16i^2$

$= 4 - 16i + 16(-1)$

$= \boxed{-12 - 16i}$

2) $w^2 = (2-3i)^2$

$= (2-3i)(2-3i)$

$= 4 - 6i - 6i + 9i^2$

$= 4 - 12i + 9(-1)$

$= \boxed{-5 - 12i}$

3) $\frac{w}{z} = \frac{2-3i}{-2+4i}$

$= \frac{(2-3i)(-2-4i)}{(-2+4i)(-2-4i)} = \frac{-4 - 8i + 6i + 12i^2}{4 + 8i - 8i - 16i^2}$

$= \frac{-4 - 2i + 12(-1)}{4 - 16(-1)}$

$= \frac{-16 - 2i}{20}$

$= \frac{-16}{20} - \frac{2}{20}i$

$= \boxed{\frac{-4}{5} - \frac{1}{10}i}$

Re $\frac{-4}{5}$

Im $\frac{-1}{10}$

Simplify $(2-3i)^3$

$= (2-3i)(2-3i)(2-3i)$

$= (4 - 6i - 6i + 9i^2)(2-3i)$

$= (4 - 12i - 9)(2-3i)$

$= (-5 - 12i)(2-3i)$

$= -10 + 15i - 24i + 36i^2$

$= -10 - 9i - 36$

$= \boxed{-46 - 9i}$

Simplify

$(2-i)(2+i)(3-2i)(3+2i)$

$= (4 + 2i - 2i - i^2)(9 + 6i - 6i - 4i^2)$

$= (4 - (-1))(9 - 4(-1)) = (4+1)(9+4)$

$= 5 \cdot 13 = \boxed{65}$

SG 9 ✓

we start on SG 10 tomorrow